Factors Reducing Call Blocking Probability by Exploiting Erlang B &C Formulas

Hassan Ali, Hafiz Faiz Rasool, Ashiq Hussain

Abstract— Resource sharing among the telecom subscribers focuses on different occasions when the incoming request for some particular connection cannot be entertained immediately and this call request either have to wait or has to be rejected by the server. Traffic congestion in the server depends upon the statistical parameters of user behavior which varies with time. Statistical Equilibrium can solve the problem regarding traffic congestion in the server which implies that "the probability per unit time of a new call request arising is equal to the probability per unit time of an existing call terminating so the average number of successful connections remains more or less constant". But this is not practical approach, because user behavior regarding telecom traffic is random.This paper illustrates that what happened to new calls which arises at that time when server has fully occupied. What will be the probability for the rejection or acceptance of the new incoming call while hitting the server. When all servers are occupied then the new call will be rejected because no path will be available to it. . The offered traffic and carried traffic are dependent on the design of our system and user behavior is independent of the system design as it is random in nature. The different assumption about the behavior of user produce significant information for both the probability distributions first is the number of occupied server and second is the loss traffic. Our work is to develop a model and to focus on the factors which will reduce the call blocking probability during overload traffic in switching system. This model is based on Erlang Formulas; we obtain call blocking probability for each type of traffic and by showing MATLAB results and mathematical models. We will examine the actual blocking occurs in system and present the new model which reduces the call blocking probability. sharing among the telecom subscribers focuses on different occasions when the incoming request for some particular connection cannot be entertained immediately and this call request either have to wait or have to be rejected by the server. Traffic congestion in the server depends upon the statistical parameters of user behavior which varies with time. Statistical Equilibrium can solve the problem regarding traffic congestion in the server which implies that "the probability per unit time of a new call request arising is equal to the probability per unit time of an existing call terminating so the average number of successful connections remains more or less constant".

Index Terms— Statistical Equilibrium, Traffic congestion, Molina equation, Erlang-B or Blocked Call Lost (BCL), Erlang-C or Blocked Call Wait (BCW), Molina Equation or Blocked Call Held (BCH)

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1 Introduction

Switching systems undergoes many problems regarding telecom traffic. There are different conditions which affects the performance of the switching systems. When there is finite number of sources, the new incoming call offered to the system can be affected by the calls which are currently in progress in the system, and when system is fully occupied, then new incoming call undergoes in blocking state which results in CONGESTION. When the number of sources are large the probability of new call arising is independent of the number of call in progress [1]. In the circuit switched systems when voice traffic is taken into account, incoming calls from the terminals are not statistically independent for 100% success rate because they experiences congestion when the server is being well occupied.

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We know that when N approaches to infinity [1], then binomial distribution approaches to Poisson distribution.

$$\binom{N}{K}\left(\frac{A}{N}\right)^{K}\left(1-\frac{A}{N}\right)^{N-K} = \frac{A^{K}}{\Gamma(K+1)}e^{-A} \quad \text{......... (1)}$$

This means that $\left(\frac{A}{N}\right)^{K}$ approaches to zero as N approaches to infinity. It shows that probability of new call originating will be decreased significantly due to congestion [1].

According to congestion theory, the value of the offered traffic exceeds then the rated value of the server which experiences congestion as no new call accepted due to the fact of the server is fully occupied. There are two ways of congestion for the blocking. first is TIME congestion, which is the percentage of time that all servers in a group are busy. Second is CALL or DEMAND congestion which is proportion of call arising which do not find a free server [2].

When the telecom traffic is overloaded due to the large number of hits on the server, we consider three possibilities to assess that particular state. These possibilities are [1]

(a) Blocked Calls Held
(b) Blocked Calls Lost
(c) Blocked Calls Wait

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In BCH, the blocked call continuously demands for service for duration of the holding time, and wishes to get successful by hunting the available slot if in this duration the server becomes free. But on the other hand this call will not be considered as a 100% successful call because some or most of the part of that particular call request will be eliminated or lost due to the overlap of the other call requests during that time period. For example the Fig 1 below [3]

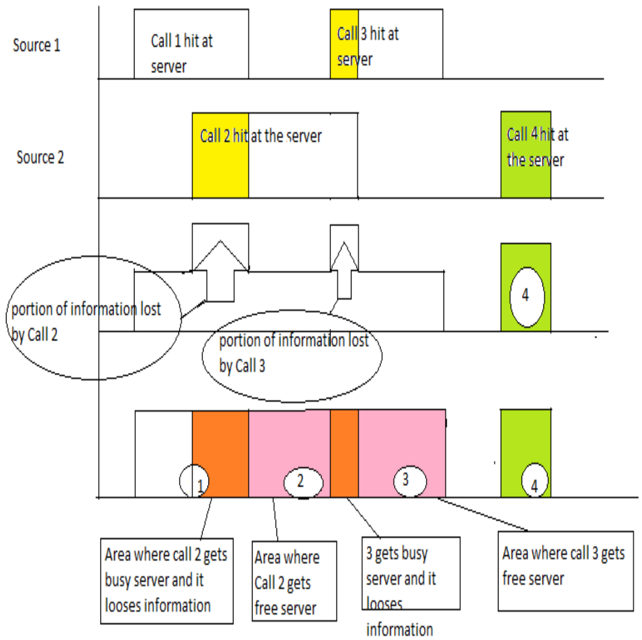

Fig 1. Blocked Calls Held [3]

Blocked calls are held by the system and serviced when the necessary facility become available, each arrival require service for a continuous period of time as shown in Fig 1 and terminates request independently when service is unavailable due to congestion. We use MOLINA equation to find held in block calls [4].

In Blocked Call Lost (BCL) scenario, the call request hits the server in the particular time when the server is busy. So the call do not avail the service and the subscriber hangs up the call so call is cleared [5]. To find blocked calls lost we use Erlang-B formula that gives probability of blocked call lost in the system due to the unavailability of the server at any instant of time. For the clear understanding of BCL, Fig 1 can be taken into account again in which call 2 will be completely lost and cleared from the system, because it does not find slot at the specific time. It is because when call 2 hit the server, at that time call 1 was still in progress [6],[7],[8].

In blocked call wait (BCW) scenario, switching system has some property through which blocked call can wait until server becomes free. The delay probability in blocked call wait is calculated by Erlang-C formula. In delay system there are finite numbers of sources. Waiting calls are selected on queuing system and use the fact that first come first served or first in first out. The blocking probability and delay probability is based on queue size in comparison with the number of effective sources [1],[8],[9].

The GOS is called call congestion or lost probability and the blocking probabilities are called time congestion. Through Matlab Results, we find the statistical characteristics of blocking in switching system.

2 Review of the State of Art

Our scope of work is the careful analysis of three scenarios, BCH, BCL and BCW by Matlab simulations and try to find out some specific traffic mathematical model, which can be the key factor to reduce the blocking probability of incoming call requests on the server. Researchers trying their level best to make such a system which may give an output of 100% successful call rate. But in practice this is not possible due to some factors like Transmission Medium, Server occupancy & its Design and Random User Behaviour regarding using the service during making their call requests.

3 Research Question and Problem Statement

When server is fully occupied then incoming call is blocked due to the server busy. So if call is held or lost then there will be loss of Erlangs. This will also reduce the efficiency of the system and will also reduce faith of consumer of such unreliable service. This problem can be overcome by two ways.

1. If we use the scheme that the number of servers and users are equal then no blocking occurs. But this is so expensive and difficult to implement.
2. If the switching system has some mechanism through which block call can wait until server becomes free. In this scheme there will be no or less loss of Erlangs.

Our research question is to identify and the cure of such factors which plays main role for the failure of incoming call requests and block them not to enter in the system, which reduces the efficiency of the system.

Moreover our research question is to find the cost at which any call that hits the server when it is fully occupied and what will be the mathematical model by the help of which that specific call will not discarded from the system.

3.1 Blocked Call Held

We have to visualize the Molina Equation, which that that " When there are large number of sources when N approaches to infinity [1], then binomial distribution approaches to Poisson distribution.

$$BT = \sum_{x-M}^{\infty} \frac{A^x}{\Gamma(x+1)} e^{-A} \quad \text{.....................} (2)$$

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Where
A = the total traffic offered in units of erlangs
N = the number of servers

During the Matlab simulation, we analyze the Molina equation with 20 call requests and with 0.5 erlang traffic intensity.

Fig 2 shows, in the start calls has highest value of being served, when number of calls increases, then system starts showing business level and in this condition the availability of free server decreases as shown in figure. In result more requests will decrease the call success rate and BCH probability will be increased. Incoming will find no server or some portion of time in the server and will be held, blocked or served with too much lost information as shown in Fig 1, call 2 and call 3.

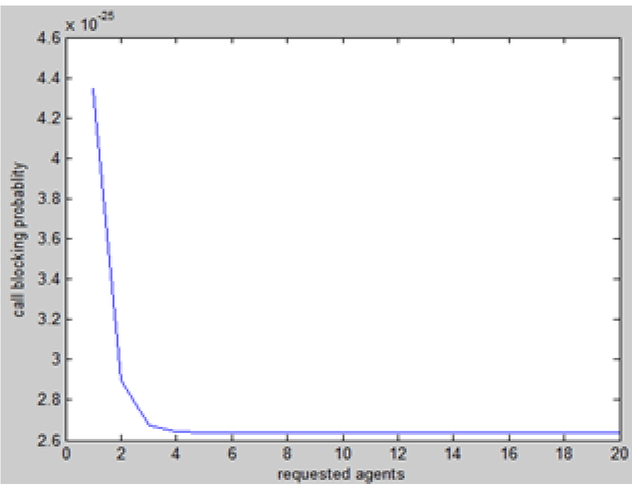

Fig 2. BCH (Molina Equation)

3.2 Blocked Call Lost

Call arises during the time when server is busy so the call do not avail the service and the subscriber hangs up the call so call is cleared. To find Block call lost we use ERLANG-B formula that gives probability of call lost in blocking of call for a system and given by [8],[10]

$$B = \frac{A^N}{\Gamma(N+1)\sum_{K=0}^{N}\left(\frac{A^K}{\Gamma(K+1)}\right)} \quad \text{.........................} (3)$$

Where:
B=Blocking probability
N=Number of servers in system
A=Offered traffic

Erlang-B formula also known as the Erlang loss formula is formula use to find the blocking probability to describe the probability of call which are loss in the system. This formula tells that when circuit is fully occupied then new call will be rejected and lost. Erlang B formula has following assumptions:

1. Infinite number of sources
2. Calls arrivals are random
3. Calls are served in order of arrival.
4. Blocked calls are lost.

During the Matlab simulation, we analyze Erlang-B equation with 20 call requests and with 0.5 erlang traffic intensity.

We analyze the Fig 3 and found the abrupt change in the graph as the call requests increases on the server. We found that initially the blocking probability approaches to zero and as soon as call requests hits the server, it gets busier and new call requests blocking probability increases. After 20 hits on the server, 21st hit will experience the blocking and that call will be cleared and will not be entertained by the system.

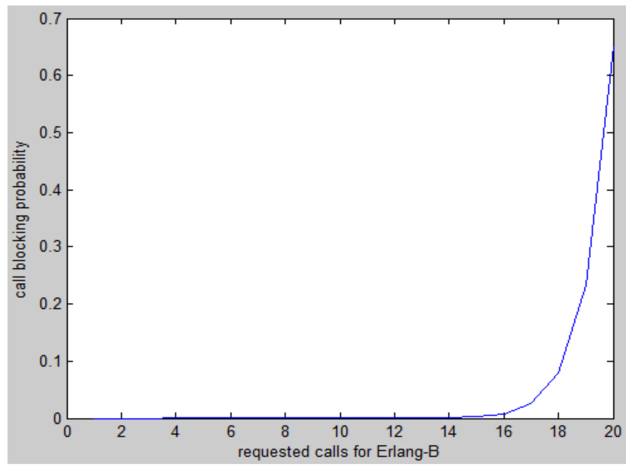

Fig 3. BCL (Erlang-B)

3.3 Blocked Call Wait

In BCW, Switching system exhibits a mechanism through which blocked calls can wait until server becomes free. In delay system there are finite numbers of sources. For the same queue, waiting calls are selected on queuing system and use the fact that first come first served or FIFO. For different queue system we use LIFO or SIRO (service in random order) mechanisms. The blocking probability and delay probability is based on queue size in comparison with the number of effective sources. The delay probability in blocked call wait is calculated by Erlang-C formula [1],[11].

η= A/M, Average Occupancy of Server
B= Blocking Probability.
A= offered Traffic.
N=M= Number of servers in the system

$$B = \frac{A^M}{\Gamma(M+1)}\frac{1}{(1-\eta)} / \sum_{x=0}^{M-1}\frac{A^x}{\Gamma(x+1)} + \frac{A^M}{\Gamma(M+1)}\frac{1}{(1-\eta)} \quad \text{............} (4)$$

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When N servers are fully occupied and the call request hits the server, the delay systems performs its duty and places the call in queue unless and until the call finds free server [12].

The Erlang C formula tells the waiting probability in a queuing system. Let's we have k number of calls and N number of servers. When K ≤ N then all the servers are not occupied and calls are not in waiting and when K > N then K - N calls will go in waiting scenario and will not dropped. This is the main merit of Erlang-C formula.

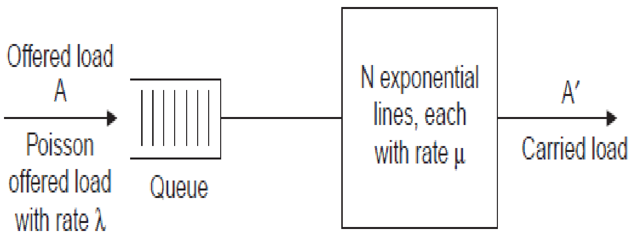

Fig (4) Queuing Model [12]

During the Matlab simulation, we analyze Erlang-C equation with 20 call requests. We analyze the Fig 5 and found that the blocking probability of call requests on the server when it is completely occupied are less than the blocking of call requests for Erlang-B formula.

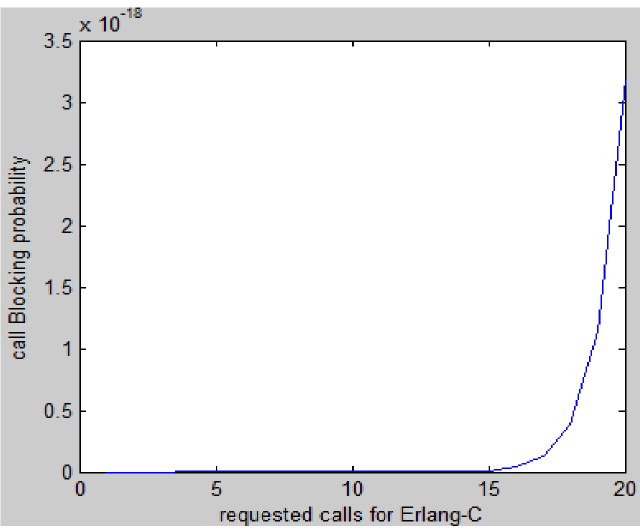

Fig (5) BCW (Erlang-C)

We found that on the usage of 0.5 Erlang by the calls, the server exhibits less blocking probability for the requested calls in Erlang-C formula the in Erlang-B formula.

4 Problem Solution

By the keen analysis of Molina equation, Erlang-B and Erlang-C formulas, it is observed that there are some key factors which behaves very important role for reducing the blocking probability of the requested calls on fully occupied servers. We analyzed these factors separately and developed an enhanced mathematical model by the help of which we can understand that what happens to the very next call request, when it hits on the fully occupied server while using Erlang- C formula.

In result, I formulate different mathematical models and proposed a model for the very next call request P(o), which indicates clear reduced blocking probability for that specific call.

By visualizing LCC (lost calls cleared) system [12], we know that:

$$P_{(K)} = \frac{1}{\Gamma(K+1)}\left(A^K\right)P(o) \quad \text{.................. (5)}$$

Where K= 1,2,3,------------------N. [12]

By the help of equation (5), I formulate the mathematical model for P(o), which is as follows.

$$\sum_{x=0}^{N-1}\left(\frac{A^x}{\Gamma(x+1)}\right) + \frac{N}{N-A}\frac{A^N}{\Gamma(N+1)} = \frac{N}{N-A}\sum_{K=0}^{N}\frac{A^K}{\Gamma(K+1)} \quad \text{...... (6)}$$

$$\frac{N-A}{N}\sum_{x=0}^{N-1}\frac{A^x}{\Gamma(x+1)} + \frac{A^N}{\Gamma(N+1)} = \sum_{K=0}^{N}\frac{A^K}{\Gamma(K+1)} \quad \text{......(7)}$$

$$\frac{1}{\frac{N-A}{N}\sum_{x=0}^{N-1}\frac{A^x}{\Gamma(x+1)} + \frac{A^N}{\Gamma(N+1)}} = \frac{1}{\sum_{K=0}^{N}\frac{A^K}{\Gamma(K+1)}} \quad \text{..........(8)}$$

Then we know that:

$$P(0) = \frac{1}{\sum_{K=0}^{N}\frac{A^K}{\Gamma(K+1)}} \quad \text{.............................. (9)}$$

Then,

$$P(o) = \frac{1}{\left(\frac{N-A}{N}\right)\sum_{x=0}^{N-1}\frac{A^x}{\Gamma(x+1)} + \frac{A^N}{\Gamma(N+1)}} \quad \text{........... (10)}$$

This is the required first call which hits the server when it is fully occupied.

Simulation results clearly shows in Fig 6, that when p(o) hits the server while using Erlang-C mechanism, the server does not clears the call but put it in queue and process it when there is free slot available. The graph also shows that reducing blocking probability for the call P(o) and for the successors of P(o) when they hits the fully occupied server.

5 Simulation Results

During the MATLAB implementation, we analyze our proposed mathematical equation with 20 call requests.

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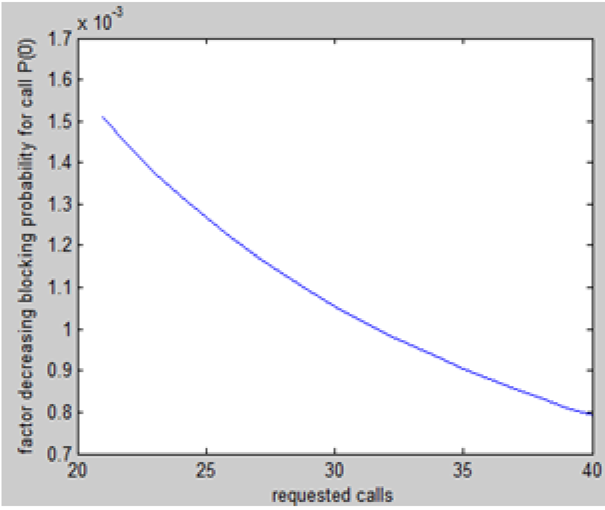

Fig (6) Proposed Model for Call P(o)

6 Conclusion

After the keen analysis of situation in which the number of servers and users are equal then no blocking occurs. But this is so expensive and difficult to implement. From Matlab results we have find that Erlang C has less blocking probability than Erlang B because the calls are in wait system and they goes in queue if the server is fully occupied. When server is fully occupied then incoming call is blocked due to the busy server. So if call is held or lost then there will be loss of Erlangs. This will also reduce the efficiency of the system and will also reduce faith of consumer of such unreliable service.

We proposed a mathematical model, by the help of which that specific call will not discarded from the system in case of fully occupied server. We proposed the reduced call blocking factor such as P(0) which shows that when using Erlang-C formula, the server does not clears the call but put it in queue and process it when there is free slot available. The proposed Mathematical Model is also defended with Matlab simulations.

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